

Sport Strategy Optimization in Beach Volleyball– How to bound direct point probabilities dependent on individual skills

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Abstract

Recently, we presented a two scale approach that uses Markov Decision problems (MDPs) to answer sport strategic questions. We have implemented our method for beach volleyball by developing an appropriate gameplay-MDP and strategic-MDP for a certain strategic benchmark question. Following the two scale approach, the gameplay-MDP is simulated to generate the input probabilities of the strategic-MDP. The strategic-MDP is solved subsequently to answer the sport strategic question. We want to investigate in this paper whether the strategic-MDP probabilities can be directly computed from the gameplay-MDP or whether at least some bounds can be computed.

The derived bounds of this paper are applied to men's beach volleyball Olympic final 2012 between Germany and Brazil and are part of the presentation *Strategy optimization in beachvolleyball – applying a two scale approach to the olympic games*.

1 Introduction

Markov Decision Problems (MDPs) can be used for modelling sport games and answering sport strategic questions. Some examples are: Clarke and Norman (2012) as well as Nadimpalli and Hasenbein (2013) investigate a Markov Decision Problem (MDP) for tennis games to determine when a player should challenge a line call. Hirotsu and Wright (2002) model football as a four state Markov Process and use dynamic programming to determine the optimal timing of a substitution Hirotsu and Wright (2002), the best policy for changing the configuration of a team Hirotsu and Wright (2003) or to determine under which circumstances a team may benefit from a professional foul Wright and Hirotsu (2003). Chan and Singal (2016) use an MDP to compute an optimization-based handicap system for tennis. Clarke and Norman (1998) formulate an MDP for cricket to determine whether the batsman should take an offered run when maximizing the probability that the better batsman is on strike at the start of the next over. Norman (1985) builds a more aggregated MDP for tennis games to tackle the question when to serve fast or when to serve slow at each stage of a game.

All papers mentioned so far investigate MDPs for general rules that are independent of teams and matches. Only a few papers present MDPs that model strategies dependent on special pairings. Most of these team specific MDPs are retrospective (Terroba et al., 2013). This comes from the difficulty to estimate appropriate transition probabilities for matches or pairings that may have not been played before. We have overcome this difficulty by our two scale approach. The idea is that the gameplay-MDP (g-MDP) incorporates only player depended probabilities that constitute the player's skills and are independent of the opponent. From a simulation of the g-MDP the transition probabilities of a second more aggregated MDP, the strategic-MDP (s-MDP) are generated. The s-MDP and the g-MDP must

be related to each other such that a set of transitions in the g-MDP can be mapped to transitions in the s-MDP. The generated s-MDP transitions depend in contrast to the g-MDP transitions on the opponent. Due to the aggregation, the s-MDP is significantly smaller than the g-MDP and can be solved by dynamic programming.

The question may arise whether the s-MDP transition probabilities can be directly computed from the g-MDP probabilities or whether at least some bounds can be found. This paper considers as a basis the implementation of the two scale approach to beach volleyball, presented in (Hoffmeister and Rambau, 2017).

The main result of this paper is the computation of intervals for the direct point probabilities of the serving situation of the Olympic beach volleyball final in 2012. The computed intervals are

$$p_{estimatedStrat}^{serve} \in [0, 0.2291], \quad \bar{p}_{estimatedStrat}^{serve} \in [0, 0.1936], \quad \hat{p}_{estimatedStrat}^{serve} \in [0.5913, 1],$$

where $p_{estimatedStrat}^{serve}$ denotes the probability for an ace, $\bar{p}_{estimatedStrat}^{serve}$ is the probability for a point loss and $\hat{p}_{estimatedStrat}^{serve}$ is the probability for a subsequent field attack by the opponent team. For the computation of these values we used the skills estimated from all prefinal matches and the estimated strategy of the final.

The paper provides the analytical derivation of the presented bounds on the direct point probabilities of the Olympic beach volleyball final in 2012 in the presentation *Strategy optimization in beachvolleyball – applying a two scale approach to the olympic games* and is organized as follows. In Section 2 the two scale approach implementation is recapped. The analytical bounds for the s-MDP transitions probabilities in terms of the g-MDP transition probabilities are derived in Section 3. Section 4 concludes this paper and gives an outlook to future investigations.

2 Two Scale Approach for Beach Volleyball recapped

In our implementation of the two scale approach for beach volleyball we answer the strategic question which attack plan a team should play, depending on the current score and situation. In beach volleyball a team consists of two players and a match of two or three sets. A set is won if one team has gained at least 21 points and is at least two points ahead of the opponent team. A point is scored according to the rally-point system. Let in the following of this paper team P be the team whose policy should be optimized and team Q be the opponent team. In both MDPs team P and team Q are modelled symmetric. However, team Q is part of the environment and captured in the transition probabilities.

The s-MDP models a complete beach volleyball set. For the purpose of the benchmark question a state of the s-MDP contains the current score, which team starts the next attack plan and an indicator whether it is a serving state or not. The action set A is constituted by the set of attack plans of team P . Attack plans consist of a sequence of hits and moves played in a phase of ball possession, e.g. an attack plan for a field attack after a serve consists of a reception, a set and a smash or shot. In all states where team P starts the next attack plan it can choose an action $a \in A$ for its next attack. The reward is modelled such that the expected total reward in a state equals the winning probability of the set starting from the current state. The transition probability p_a [\bar{p}_a] is the probability that team P playing action a directly wins [loses] the rally. The probability that none of this happens is denoted by $\hat{p}_a := 1 - p_a - \bar{p}_a$. We use q, \bar{q} and \hat{q} analogously for the transition probabilities after an serving or field attack of team Q . Since a serving attack has transition probabilities clearly different from a field attack, we distinguish between them. This is denoted by a superscript *field* or *serve* on the transition probabilities. Thus the evolution of the system is governed by eight probabilities p_a^{sit} , \bar{p}_a^{sit} , q^{sit} , \bar{q}^{sit} , where $a \in A$, $sit \in \{serve, field\}$.

The g-MDP models only a single rally instead of a whole set. A state includes the position of each player, the position of the ball, a boolean variable that indicates the hardness of the last hit and three other parameters that are necessary to track certain beach volleyball rules. A position on the court is defined on basis of the grid presented in Figure 1. The g-MDP is very large and contains around one

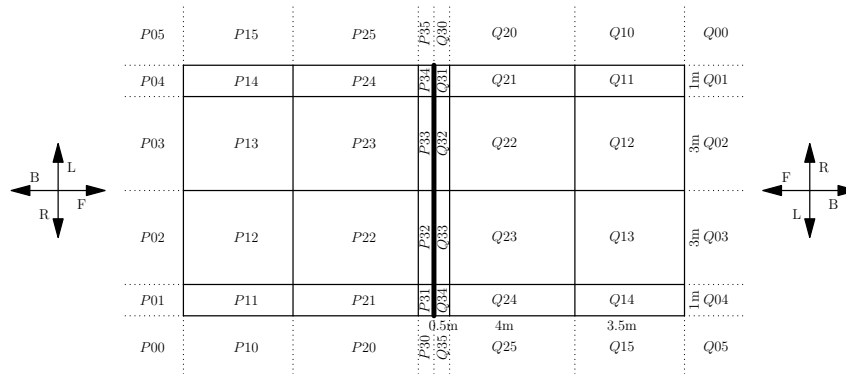


Figure 1: Court grid

billion different states. A state in the g-MDP is observed each time a player hits the ball, or the ball contacts the ground or net. The important point is that all transitions are generated from the individual player skills. The advantage of the player skills is that they can be estimated from training sessions or any match of that player. Furthermore, the player skills are assumed to be more stable in comparison to opponent dependent probabilities.

For each player ρ , including the opponent team, and each hitting technique $tech$ with target $target$ against a ball with hardness $hardness$, the probability $p_{succ, \rho}(tech, [pos(\rho)], [target], [hardness])$ is defined as the probability that the specified target field $target$ from ρ 's position is met. Due to modelling issues of the g-MDP implementation, we distinguish skills depending on parameters specified in the curly brackets. The parameters in square brackets are optional parameters. For some hitting techniques, the individual player probabilities are aggregated over certain optional parameters. For example, serving skills are aggregated over all possible serving fields which are $P01, P02, P03$ and $P04$. Since only the relative distance to the ball is important for receives, defences and settings, they are aggregated over all possible target fields. But, we distinguish if a defence or receive was played against a *normal* or *hard* ball which is indicated by the parameter $hardness \in \{hard, normal\}$. If in the following some optional parameters are missing, it means that the skill was aggregated over that parameter. The complete tables with the estimated individual player skills and their aggregation can be found in (Hoffmeister and Rambau, 2017, Section 7, Gameplay MDP validation). We want to point out that a successful hit does not mean that a point occurs, since this would include the defending or receiving skills of the opponents. It means that the ball flies toward the target field the player aimed for. The probability of an execution fault is denoted by $p_{fault, \rho}(tech, [pos(\rho)], [target], [hardness])$ for player ρ using hitting technique $tech$. If neither an execution fault nor a successful hit occurs, the ball will land in a neighbour field of the target field. We call this event a deviation and denote it by the probability $p_{dev, \rho}(tech, [pos(\rho)], [target], [hardness]) := 1 - p_{succ, \rho}(tech, [pos(\rho)], [target], [hardness]) - p_{fault, \rho}(tech, [pos(\rho)], [target], [hardness])$. Table 1 summarizes all hitting techniques available in the g-MDP. The possible target fields $neighbour(pos(\rho)) \setminus (Q, \cdot)$ of a set are all fields that are a neighbouring field of the player's current position $pos(\rho)$ and not on team Q 's court. There exist requirements on the state for using a certain technique which we skipped here and are specified in (Hoffmeister and Rambau,

<i>tech</i>	<i>target</i>	description	results in <i>hard</i> ball	skill depends on <i>hardness</i>
Serve				
S_F	$Q11 - Q24$	float serve	false	false
S_J	$Q11 - Q24$	jump serve (hard)	true	false
Reception				
r	$P11 - P34$	receive	false	true
r_m	$P11 - P34$	receive with move	false	true
Setting				
s	$neighbour(pos(\rho)) \setminus (Q, \cdot)$	set	false	false
Attack-Hit				
F_{SM}	$Q11 - Q24$	smash (hard)	true	false
F_E	$Q11 - Q24$	emergency shot	false	false
F_P	$Q11 - Q34$	planned shot	false	false
Defence				
d	$P11 - P34$	defence	false	true
d_m	$P11 - P34$	defence with move	false	true

 Table 1: Hit specification for player ρ of team P and ball $ball$;

2017, see Section 5, A Gameplay MDP for Beach Volleyball). We included the information which hitting technique results in a *hard* ball in the table.

Team actions in the g-MDP are a composition of the players' moves and hits. Each team in the g-MDP plays a team specific g-MDP-policy that is an implementation of an s-MDP action in the g-MDP. We implemented a g-MDP-policy as a variation of a basic policy that guarantees a reasonable match flow. Each team adapts parts of the basic policy according to their preferences. In our implementation modifications of the blocking, serving and attack-hit decisions are possible. All team specific refinements are included in a vector π whose components determine the probability for choosing true in a Boolean decision. In the basic policy all components of π are set to 0.5 which means, that both decision possibilities are equally probable. The blocking policy is specified by π_b , which states with which probability player 1 of a team is the designated blocking player in the next rally. It follows that with probability $(1 - \pi_b)$ player 2 is the blocking player. The parameter π_s determines the serving policy of a team. With π_s a serve on player 1 of the opponent team is made, i.e., the target field of the serve belongs to the opposing court half that is covered by player 1. Further, a technique and target field decision of the serve and attack-hit are included in π_h . The two parts π_h^{serve} and π_h^{field} of π_h include the policy belonging to the indicated situation. Each part splits up into a technique and target field decision that depend on the hitting player ρ , i.e., $\pi_h^{sit} = (\pi_{h,tech}^{sit}(\rho), \pi_{h,target}^{sit}(\rho))^T$ with $sit \in \{serve, field\}$. The subscript term indicates if the decision is related to the technique (*tech*) or target field (*target*) decision. Now we can summarize all parameters that are necessary for defining a g-MDP policy of team P with players P_1 and P_2 :

$$\pi = \begin{pmatrix} \pi_h \\ \pi_b \\ \pi_s \end{pmatrix}, \quad \pi_h = \begin{pmatrix} \pi_h^{serve} \\ \pi_h^{field} \end{pmatrix}, \quad \pi_h^{sit} = \begin{pmatrix} \pi_{h,tech}^{sit}(\rho) \\ \pi_{h,target}^{sit}(\rho) \end{pmatrix}, \quad sit \in \{serve, field\}, \quad \rho \in \{P_1, P_2\}.$$

For a better memorability we defined the values of the components of π_h always as the probability for the more risky opportunity. In our example, we have two serving techniques available in the g-MDP, namely the float serve S_F and the jump serve S_J . The float serve is considered as a safe hit and the jump serve as a

risky hit. So $\pi_{h,tech}^{serve}(\rho)$ is defined as the probability that ρ chooses an S_J . For the attack-hit we have three techniques available the smash F_{SM} , a planned shot F_P and an emergency shot F_E . The emergency shot is normally only played if none of the other attack-hits is possible. The smash is considered as a risky hit and the planned shot as a safe hit. So $\pi_{h,tech}^{field}(\rho)$ is defined as the probability that ρ chooses a F_{SM} . Furthermore, we define all fields that are near the side out of the court as border fields. For example, on court side of team Q the border fields are $\partial F := \{Q11 - Q31, Q14 - Q34\}$. These are more risky target fields than non-border fields. So $\pi_{h,target}^{serve}(\rho)$ and $\pi_{h,target}^{field}(\rho)$ are the probabilities with which a border field is chosen as a target field. (Hoffmeister and Rambau, 2017, see Section 6, Gameplay MDP strategy)

Following the two scale approach, the g-MDP is simulated with team P playing a certain policy. From the simulation the s-MDP transition probabilities are estimated by counting the number of serves and attack plans as well as their outcomes. The outcome, following the definition of the s-MDP, is either a direct point or fault of the attacking team or a subsequent attack by the opponent team.

3 Bounds for the Direct Point Probabilities

In the following we derive bounds for the s-MDP probabilities in terms of the g-MDP probabilities. We make these considerations for team P 's probabilities only. Bounds for the opponent team Q 's probabilities can be derived analogously. For easier notation, we denote the hitting player throughout this section by $\rho \in \{P1, P2\}$ and the receiving Player by $\sigma \in \{Q1, Q2\}$. Further, let $S_* [r_*]$ be an unspecified serve [reception]. We assume that ρ has chosen an reasonable target field, i.e., *target* is inside the court of the opposing team.

We start with an analysis of the serving situation, i.e., we compute bounds for p_a^{serve} , \bar{p}_a^{serve} and \hat{p}_a^{serve} , where a is an attack plan of the s-MDP that corresponds to a team specific policy π of the g-MDP. In beach volleyball a direct point after a serve is called an ace. For an ace the following events must all be realized together in a serving situation of the g-MDP:

- the serve is executed without a fault,
- the ball does not land in an outside field,
- the opponent team makes a fault when receiving the ball.

Now we try to calculate the probabilities of these events. Assume first, the hitting player ρ , the executed serving technique S_* and the target field *target* are known. Then the probability that S_* is executed by ρ without a fault is

$$p_{succ, \rho}(S_*, target) + p_{dev, \rho}(S_*, target)$$

Since we know from policy π of team P with which probability the serves S_J or S_F are played we can state that expression more precisely as

$$\begin{aligned} & \pi_{h,tech}^{serve}(\rho) \cdot \left(p_{succ, \rho}(S_J, target) + p_{dev, \rho}(S_J, target) \right) \\ & + (1 - \pi_{h,tech}^{serve}(\rho)) \cdot \left(p_{succ, \rho}(S_F, target) + p_{dev, \rho}(S_F, target) \right) \end{aligned}$$

The hit may only land outside the field if the target field was a border field and a deviation to an outside field occurred. From the policy π the probability $\pi_{h,target}^{serve}(\rho)$ with which a border field is chosen as a target field is known. In the system dynamic of the g-MDP a deviation to any neighbour-field is equally probable. When we look at the court grid presented in Figure 1, we see that each field has eight neighbour-fields and the number of outside fields that are neighbour-fields range between three and five.

These values depend on the specification of the court grid. In general, let

$$\omega(\text{target}) := \frac{|\text{neighbour}(\text{target}) \cap \text{outside-fields}(\text{target})|}{|\text{neighbour}(\text{target})|}$$

be the probability that the result of the deviation from field target is an outside field. Since we do not know the exact target field, we define

$$\begin{aligned} \omega_{\max} &= \max \left\{ \frac{|\text{neighbour}(\text{target}) \cap \text{outside-fields}(\text{target})|}{|\text{neighbour}(\text{target})|} \mid \forall \text{target} \in \text{grid} \right\} \\ \omega_{\min} &= \min \left\{ \frac{|\text{neighbour}(\text{target}) \cap \text{outside-fields}(\text{target})|}{|\text{neighbour}(\text{target})|} \mid \forall \text{target} \in \text{grid} \right\} \end{aligned}$$

For our grid of the g-MDP, we get $\omega_{\max} = \frac{5}{8}$ and $\omega_{\min} = \frac{3}{8}$.

The receiving player σ may use, depending on his position and the position of the ball, a receive r or a receive with a move r_m as the receiving technique. For receiving and defending skills, the individual probabilities for that technique depend also on the *hardness* of the ball which may be either *hard* or *normal*. The jump serve is the only serve that leads to a *hard* ball. Since we know from policy π of the attacking team the probability of a jump serve which is $\pi_{h,tech}^{serve}(\rho)$ we know with which probability the ball to receive is *hard* or *normal*. The absolute position of the receiving player has in the g-MDP no impact on the skills. So σ makes an execution fault with probability

$$\pi_{h,tech}^{serve}(\rho) \cdot p_{\text{fault}, \sigma}(r_*, \text{normal}) + (1 - \pi_{h,tech}^{serve}(\rho)) p_{\text{fault}, \sigma}(r_*, \text{hard}).$$

Since in general a receive with a move should have a higher fault rate than a receive without a move, we can conclude that the probability of a fault lies between

$$\pi_{h,tech}^{serve}(\rho) \cdot p_{\text{fault}, \sigma}(r, \text{hard}) + (1 - \pi_{h,tech}^{serve}(\rho)) p_{\text{fault}, \sigma}(r, \text{normal})$$

and

$$\pi_{h,tech}^{serve}(\rho) \cdot p_{\text{fault}, \sigma}(r_m, \text{hard}) + (1 - \pi_{h,tech}^{serve}(\rho)) p_{\text{fault}, \sigma}(r_m, \text{normal}).$$

It depends on the development of the match which player serves how often. Also we have no stochastic information about the serving distribution. Therefore, we introduce $\rho_{\max}^{\gamma, tech}$ [$\rho_{\min}^{\gamma, tech}$] as the player that has the maximal [minimal] probability for the specified outcome γ of a skill, i.e., we chose

$$(\rho_{\max}^{\gamma, tech}, \tau_{\max}^{\gamma, tech}) \in \arg \max_{\rho, \text{target}} \left\{ p_{\gamma, \rho}(\text{tech}, \text{target}) \mid \rho \in \{P1, P2\} \right\} \forall \gamma \in \{\text{succ}, \text{fault}, \text{dev}\}, \text{tech} \in \{S_J, S_F\}$$

and

$$(\rho_{\min}^{\gamma, tech}, \tau_{\min}^{\gamma, tech}) \in \arg \min_{\rho, \text{target}} \left\{ p_{\gamma, \rho}(\text{tech}, \text{target}) \mid \rho \in \{P1, P2\} \right\} \forall \gamma \in \{\text{succ}, \text{fault}, \text{dev}\}, \text{tech} \in \{S_J, S_F\}.$$

So $(\rho_{\max}^{\gamma, tech}, \tau_{\max}^{\gamma, tech})$ or $(\rho_{\min}^{\gamma, tech}, \tau_{\min}^{\gamma, tech})$ can be for the same hit a different player depending on the specified outcome γ . We use an analogous notation for the receiving player σ . The components of the hitting strategy π_h are dependent on the hitting player. Therefore, we define

$$\begin{aligned} \bar{\pi}_{h,tech}^{serve} &= \max_{\rho \in \{P1, P2\}} \{ \pi_{h,tech}^{serve}(\rho) \} \quad \text{and} \quad \underline{\pi}_{h,tech}^{serve} = \min_{\rho \in \{P1, P2\}} \{ \pi_{h,tech}^{serve}(\rho) \} \\ \bar{\pi}_{h,target}^{serve} &= \max_{\rho \in \{P1, P2\}} \{ \pi_{h,target}^{serve}(\rho) \} \quad \text{and} \quad \underline{\pi}_{h,target}^{serve} = \min_{\rho \in \{P1, P2\}} \{ \pi_{h,target}^{serve}(\rho) \} \end{aligned}$$

We can summarize the computed bounds for p^{serve} :

$$\begin{aligned}
 p_a^{serve} \leq & \left(\bar{\pi}_{h,tech}^{serve} \cdot \left(p_{succ, \rho_{max}^{succ}, S_J} (S_J, \tau_{max}^{succ}, S_J) + p_{dev, \rho_{max}^{dev}, S_J} (S_J, \tau_{max}^{succ}, S_J) \right. \right. \\
 & \cdot \left. \left. \left((1 - \underline{\pi}_{h,target}^{serve}) + \bar{\pi}_{h,target}^{serve} (1 - \omega_{min}) \right) \right) \right. \\
 & + \left. (1 - \underline{\pi}_{h,tech}^{serve}) \left(p_{succ, \rho_{max}^{succ}, S_F} (S_F, \tau_{max}^{succ}, S_F) + p_{dev, \rho_{max}^{dev}, S_F} (S_F, \tau_{max}^{succ}, S_F) \right. \right. \\
 & \cdot \left. \left. \left((1 - \underline{\pi}_{h,target}^{serve}) + \bar{\pi}_{h,target}^{serve} (1 - \omega_{min}) \right) \right) \right) \\
 & \cdot \left(\bar{\pi}_{h,tech}^{serve} \cdot p_{fault, \sigma_{max}^{fault}, r_m} (r_m, hard) + (1 - \underline{\pi}_{h,tech}^{serve}) p_{fault, \sigma_{max}^{fault}, r_m} (r_m, normal) \right)
 \end{aligned}$$

and

$$\begin{aligned}
 p_a^{serve} \geq & \left(\underline{\pi}_{h,tech}^{serve} \cdot \left(p_{succ, \rho_{min}^{succ}, S_J} (S_J, \tau_{min}^{succ}, S_J) + p_{dev, \rho_{min}^{dev}, S_J} (S_J, \tau_{min}^{dev}, S_J) \right) \right. \\
 & \cdot \left. \left((1 - \bar{\pi}_{h,target}^{serve}) + \underline{\pi}_{h,target}^{serve} (1 - \omega_{max}) \right) \right) \\
 & + \left(1 - \bar{\pi}_{h,tech}^{serve} \right) \left(p_{succ, \rho_{max}^{succ}, S_F} (S_F, \tau_{min}^{succ}, S_F) + p_{dev, \rho_{min}^{dev}, S_F} (S_F, \tau_{min}^{dev}, S_F) \right. \\
 & \cdot \left. \left((1 - \bar{\pi}_{h,target}^{serve}) + \underline{\pi}_{h,target}^{serve} (1 - \omega_{max}) \right) \right) \\
 & \cdot \left(\underline{\pi}_{h,tech}^{serve} \cdot p_{fault, \sigma_{min}^{fault}, r} (r, hard) + (1 - \bar{\pi}_{h,tech}^{serve}) p_{fault, \sigma_{min}^{fault}, r} (r, normal) \right).
 \end{aligned}$$

In the next step we consider the probability \bar{p}_a^{serve} of a serving fault. We make the plausible assumption that the opponent does not try to receive a serve that flies towards an outside field. Each of the following events in the g-MDP lead to a fault after a serve:

- execution fault of the serve, i.e., the ball does not cross the net,
- the ball crosses the net but lands in an outside field.

In the same way as for the direct point probability, we can calculate the probability of an execution fault of the hitting player ρ when following policy π :

$$\pi_{h,tech}^{serve}(\rho) \cdot p_{fault, \rho} (S_J, target) + (1 - \pi_{h,tech}^{serve}(\rho)) p_{fault, \rho} (S_F, target).$$

Since we assume only reasonable serves, the ball can only land in an outside field if a deviation occurred. Analogously to the analysis of p_a^{serve} , we can calculate a lower bound for the probability that the ball crosses the net and lands in an outside field:

$$\left(\pi_{h,tech}^{serve}(\rho) \cdot p_{dev, \rho} (S_J, target) + (1 - \pi_{h,tech}^{serve}(\rho)) p_{dev, \rho} (S_F, target) \right) \cdot \pi_{h,target}^{serve}(\rho) \cdot \omega_{min}$$

and an upper bound:

$$\left(\pi_{h,tech}^{serve}(\rho) \cdot p_{dev, \rho} (S_J, target) + (1 - \pi_{h,tech}^{serve}(\rho)) p_{dev, \rho} (S_F, target) \right) \cdot \pi_{h,target}^{serve}(\rho) \cdot \omega_{max}.$$

With the same meaning of $\rho_{\max}^{\gamma, tech}$, $\rho_{\min}^{\gamma, tech}$, $\tau_{\max}^{\gamma, tech}$, $\tau_{\min}^{\gamma, tech}$, $\bar{\pi}_{h,tech}^{serve}$, $\underline{\pi}_{h,tech}^{serve}$, $\bar{\pi}_{h,target}^{serve}$ and $\underline{\pi}_{h,target}^{serve}$ as before, we get an upper bound for \bar{p}^{serve} :

$$\begin{aligned} \bar{p}_a^{serve} &\leq \bar{\pi}_{h,tech}^{serve} \cdot p_{fault, \rho_{\max}^{fault, S_J}}(S_J, \tau_{\max}^{fault, S_J}) + (1 - \underline{\pi}_{h,tech}^{serve}) p_{fault, \rho_{\max}^{fault, S_F}}(S_F, \tau_{\max}^{fault, S_F}) \\ &\quad + \left(\bar{\pi}_{h,tech}^{serve} \cdot p_{dev, \rho_{\max}^{dev, S_J}}(S_J, \tau_{\max}^{dev, S_J}) + (1 - \underline{\pi}_{h,tech}^{serve}) p_{dev, \rho_{\max}^{dev, S_F}}(S_F, \tau_{\max}^{dev, S_F}) \right) \cdot \bar{\pi}_{h,target}^{serve} \cdot \omega_{\max} \end{aligned}$$

and a lower bound:

$$\begin{aligned} \bar{p}_a^{serve} &\geq \underline{\pi}_{h,tech}^{serve} \cdot p_{fault, \rho_{\min}^{fault, S_J}}(S_J, \tau_{\min}^{fault, S_J}) + (1 - \bar{\pi}_{h,tech}^{serve}) p_{fault, \rho_{\min}^{fault, S_F}}(S_F, \tau_{\min}^{fault, S_F}) \\ &\quad + \left(\underline{\pi}_{h,tech}^{serve} \cdot p_{dev, \rho_{\min}^{dev, S_J}}(S_J, \tau_{\min}^{dev, S_J}) + (1 - \bar{\pi}_{h,tech}^{serve}) p_{dev, \rho_{\min}^{dev, S_F}}(S_F, \tau_{\min}^{dev, S_F}) \right) \cdot \underline{\pi}_{h,target}^{serve} \cdot \omega_{\min}. \end{aligned}$$

Finally, we compute bounds for the case that neither a direct point nor a fault occurs, which happens with probability \hat{p}_a^{serve} . The following events must occur together after a serving situation in the g-MDP to lead to a subsequent attack by the opponent team:

- the serve is executed without a fault,
- the ball does not land in an outside field,
- the opponent team is receives the ball without a fault.

The first two events are the same as in a direct point scenario. Only the receiving event differs and is the counterpart of the receiving event in the direct point scenario.

$$\begin{aligned} \hat{p}_a^{serve} &\leq \left(\bar{\pi}_{h,tech}^{serve} \cdot \left(p_{succ, \rho_{\max}^{succ, S_J}}(S_J, \tau_{\max}^{succ, S_J}) + p_{dev, \rho_{\max}^{dev, S_J}}(S_J, \tau_{\max}^{succ, S_J}) \right) \right. \\ &\quad \cdot \left. \left((1 - \underline{\pi}_{h,target}^{serve}) + \bar{\pi}_{h,target}^{serve} (1 - \omega_{\min}) \right) \right) \\ &\quad + \left(1 - \underline{\pi}_{h,tech}^{serve} \right) \left(p_{succ, \rho_{\max}^{succ, S_F}}(S_F, \tau_{\max}^{succ, S_F}) + p_{dev, \rho_{\max}^{dev, S_F}}(S_F, \tau_{\max}^{succ, S_F}) \right) \\ &\quad \cdot \left. \left((1 - \underline{\pi}_{h,target}^{serve}) + \bar{\pi}_{h,target}^{serve} (1 - \omega_{\min}) \right) \right) \\ &\quad \cdot \left(\bar{\pi}_{h,tech}^{serve} \cdot \left(p_{succ, \sigma_{\max}^{succ, r}}(r, hard) + p_{dev, \sigma_{\max}^{dev, r}}(r, hard) \right) \right) \\ &\quad + \left. \left(1 - \underline{\pi}_{h,tech}^{serve} \right) \left(p_{succ, \sigma_{\max}^{succ, r}}(r, normal) + p_{dev, \sigma_{\max}^{dev, r}}(r, normal) \right) \right) \end{aligned}$$

and

$$\begin{aligned} \hat{p}_a^{serve} &\geq \left(\underline{\pi}_{h,tech}^{serve} \cdot \left(p_{succ, \rho_{\min}^{succ, S_J}}(S_J, \tau_{\min}^{succ, S_J}) + p_{dev, \rho_{\min}^{dev, S_J}}(S_J, \tau_{\min}^{dev, S_J}) \right) \right. \\ &\quad \cdot \left. \left((1 - \bar{\pi}_{h,target}^{serve}) + \underline{\pi}_{h,target}^{serve} (1 - \omega_{\max}) \right) \right) \\ &\quad + \left(1 - \bar{\pi}_{h,tech}^{serve} \right) \left(p_{succ, \rho_{\max}^{succ, S_F}}(S_F, \tau_{\min}^{succ, S_F}) + p_{dev, \rho_{\min}^{dev, S_F}}(S_F, \tau_{\min}^{dev, S_F}) \right) \\ &\quad \cdot \left. \left((1 - \bar{\pi}_{h,target}^{serve}) + \underline{\pi}_{h,target}^{serve} (1 - \omega_{\max}) \right) \right) \\ &\quad \cdot \left(\underline{\pi}_{h,tech}^{serve} \cdot \left(p_{succ, \sigma_{\min}^{succ, r_m}}(r_m, hard) + p_{dev, \sigma_{\min}^{dev, r_m}}(r_m, hard) \right) \right) \\ &\quad + \left. \left(1 - \bar{\pi}_{h,tech}^{serve} \right) \left(p_{succ, \sigma_{\min}^{succ, r_m}}(r_m, normal) + p_{dev, \sigma_{\min}^{dev, r_m}}(r_m, normal) \right) \right). \end{aligned}$$

Figure 2 summarizes realisation sequences of the g-MDP serving situation and the related s-MDP transitions. Probability of the paths in the g-MDP that end up in a green disk correspond to the path of p_a^{serve} in the s-MDP, probabilities of paths with an orange circle correspond to \hat{p}_a^{serve} and red disks to \bar{p}_a^{serve} .

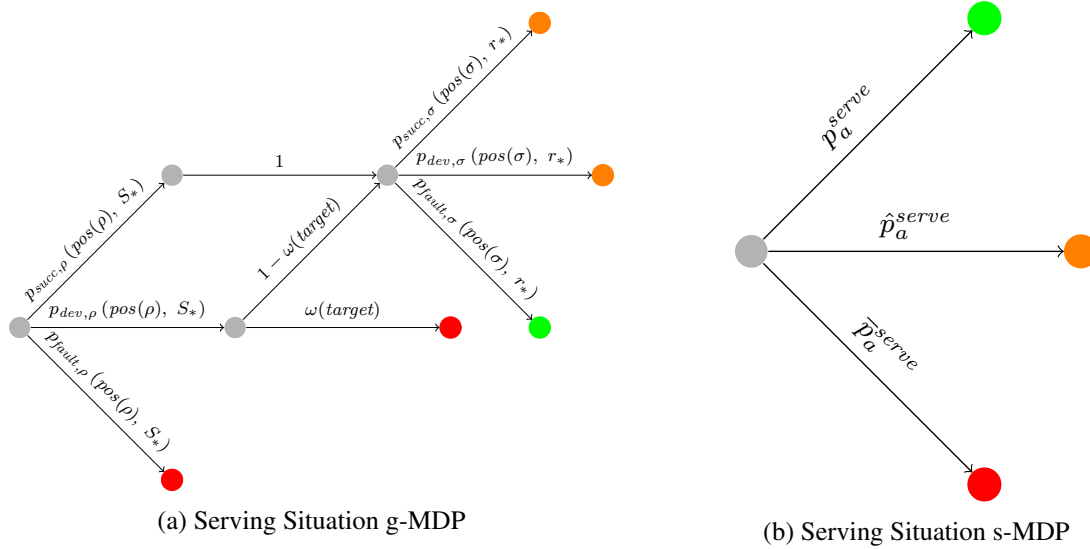


Figure 2: Summary Serving Situation

If we insert the estimated pre-final skills of Brink-Reckermann and Alison-Emanuel as well as the estimated final strategy of Brink-Reckermann in the presented equations we get the intervals for the direct point probabilities as presented in the introduction of that paper. For a comparison, the estimates for the direct point probabilities from the g-MDP simulation, presented in (Hoffmeister and Rambau, 2017), are:

$$P_{estimatedStrat}^{serve} = 0.0769, \quad \bar{P}_{estimatedStrat}^{serve} = 0.1380, \quad \hat{P}_{estimatedStrat}^{serve} = 0.7851.$$

4 Conclusion

We conclude that our estimates for the direct point probabilities from the g-MDP simulation lie in the computed intervals of this paper. The derived bounds have a relative large spread because of the large number of possible actions and realisations in the g-MDP. Since it is not possible, e.g., to predict how often a player may serve in a set or which player of the opponent team will receive the ball, we had to make rough assessments in the terms for the bounds. The computed intervals will get smaller if the players in a team have more similar skills.

It is work in progress to analyse the field attack situation in a similar way even if this will be probably even more complicated. However, we conclude that it is possible to compute some bounds of the s-MDP transition probabilities in terms of the g-MDP strategy and skills.

References

- [1] Chan, T. C. Y. and Singal, R. (2016) *A Markov Decision Process-based handicap system for tennis*. Journal of Quantitative Analysis in Sports, Vol. **12**, pp. 179–189.

- [2] Clarke, S. R. and Norman, J. M. (1998) *Dynamic programming in cricket: Protecting the weaker batsman*. Asia Pacific Journal of Operational Research, Vol. **15**.
- [3] Clarke, S. R. and Norman, J. M. (2012) *Optimal challenges in tennis*. Journal of the Operational Research Society, Vol. **63**, pp. 1765–1772.
- [4] Hirotsu, N. and Wright, M. (2002) *Using a Markov process model of an association football match to determine the optimal timing of substitution and tactical decisions*. Journal of the Operational Research Society, Vol. **53**, pp. 88–96.
- [5] Hirotsu, N. and Wright, M. (2003) *Determining the best strategy for changing the configuration of a football team*. Journal of the Operational Research Society, Vol. **54**, pp. 878–887.
- [6] Hoffmeister, S. and Rambau, J. (2017) *Strategy Optimization in Sports – A Two-Scale Approach via Markov Decision Problems*. URL: http://www.wm.uni-bayreuth.de/de/download/xcf2d3wd41kj2/preprint_sso_bv.pdf.
- [7] Nadimpalli, V. K. and Hasenbein, J. J. (2013) *When to challenge a call in tennis: A Markov decision process approach*. Journal of Quantitative Analysis in Sports, Vol. **9**, pp. 229–238.
- [8] Norman, J. M. (1985) *Dynamic Programming in Tennis – When to Use a Fast Serve*. The Journal of the Operational Research Society, Vol. **36**, pp. 75–77.
- [9] Terroba, A., Kusters, W., Varona, J. and Manresa-Yee, C. S. (2013) *Finding optimal strategies in tennis from video sequences*. International Journal of Pattern Recognition and Artificial Intelligence, Vol. **27**, pp. 1–31.
- [10] Wright, M. and Hirotsu, N. (2003) *The professional foul in football: Tactics and deterrents*. Journal of the Operational Research Society, Vol. **54**, pp. 213–221.